Adaptive Estimation and Control of MR damper for Semi-active Suspension Systems

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Abstract: This paper proposes adaptive estimation and control methods for vehicle semi-active suspension systems with magneto-rheological (MR) damper. To incorporate MR damper into the control design, a hyperbolic model is adopted to describe its dynamics, and then adaptive parameter estimation is firstly studied to estimate the unknown parameters of the hyperbolic model. This estimation method requires the measured piston variable and damper force, and can be taken as a further extension of our recently proposed parameter estimation error based algorithms. Moreover, an adaptive control is designed to stabilize the vertical vehicle displacement to improve the ride comfort, where an alternative leakage term is introduced in the adaptive law to guarantee simultaneously the precise estimation of several essential parameters (e.g. mass of vehicle body and MR damper parameters) and the control convergence. The closed-loop system stability is proved and relevant suspension performance requirements are analyzed. Finally, simulations based on a quarter-car model are provided to validate the proposed method.

Key Words: Semi-active suspension systems, magneto-rheological damper, parameter estimation, adaptive control

1. Introduction

As widely used in the semi-active control device, electro-rheological (ER) and magneto-rheological (MR) fluids have been well recognized as specific smart materials, because their rheological properties can be changed in millisecond time period by tuning the electric field or magnetic field [1]. In the vehicle suspension applications, the induced stress of the fluid should be large enough to eliminate the applied force. Compared to ER fluid, MR fluid can produce 50~100kPa stress with good controllability and small power consumption. Hence, MR fluid and the associated MR dampers have been used in vibration controls [2, 3], e.g. bridge structure, building and vehicle suspension systems [4, 5].

Vehicle suspension system is designed to improve the vehicle maneuverability, ride comfort and safety [6, 7]. Generally, the suspension system can be divided into three types: passive suspension, semi-active suspension and active suspension [8, 9]. Due to its low cost, low energy consumption and high reliability in comparison to active suspensions, semi-active suspension has attracted significant attention of both academics and engineers [2]. It is noted that the suspension system with MR dampers can be taken as semi-active suspension device because the damper force can be changed by using variable damping or energy dissipation components. However, the control of semi-active suspension with MR damper has not been fully solved since the induced hysteretic dynamics in the MR damper [10].

To accurately describe the dynamics of MR damper, several mathematical models have been proposed to capture the hysteresis and bi-viscous characteristic, such as Bingham model [11], Bouc-wen model [12] and spencer model [13], etc. However, the conflictions between the complexity and the modeling precision of MR damper are always significant in aforementioned models; for example, adopting a complex MR damper model to comprehensively describe its dynamical characteristics may make the parameter identification difficult. Thus, the tradeoff between those conflicting requirements should be addressed carefully when semi-active suspension with MR damper is used in practical systems.

In fact, application of MR damper in the vehicle suspension system has been reported in some literatures [10, 14-16]. According to magneto-rheological technology, the adjustment of the damping force can be realized by changing the input current. This in turn can eliminate the vehicle oscillation and thus improve the ride comfort and operation stability. In [17], a semi-active suspension with an LPV/ H_∞ control method was designed. In [15], an adaptive control method was designed to manage the complex hysteretic nonlinearities. However, the accurate online modeling of suspension system with MR damper deserves further investigation.

In this paper, we will incorporate the MR damper into a semi-active suspension system and present an alternative modeling and control method. First, several widely used MR damper models are reviewed, and a hyperbolic MR damper model depending on the hysteric variable and damper force is adopted. Inspired by our recent work [18, 19], we propose an adaptive parameter estimation method to online identify the unknown model parameters. Furthermore, an adaptive control is introduced for semi-active vehicle suspension systems with unknown hyperbolic MR damper. This control can regulate the vehicle vertical displacement by manipulating the applied current. With the aim to achieve simultaneous online modeling and control, a new leakage term as [19] is introduced in the adaptive law, such that the estimated parameters converge to their true values. In this case, the performance of control system can be greatly improved. The suspension performance requirements are also studied. Finally, simulation results are given to illustrate the efficacy of the proposed method.

2. Modeling of Magneto-rheological Damper

2.1 MR damper dynamics

Magneto-rheological fluid consists of ferromagnetic particles, base liquid and stabilizer. Under zero magnetic field conditions, MR fluid can present a low viscosity Newtonian fluid state. However, with the increased magnetic field intensity, the fluid

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transforms into the Bingham liquid with high viscosity and low liquidity [2, 3]. This conversion is continuous and reversible, which can occur in the millisecond time, and thus MR fluid can be taken as a kind of controllable fluid [13]. This salient feature makes it possible to use MR fluid as the working medium for constructing MR damper as a semi-active control device. This kind of MR dampers have advantages of simple structure, fast response, low power consumption, continuously adjustable and high damping force.

In the vehicle suspension system, the work process of MR damper is shown in Fig.1. The vehicle's ECU can calculate a current (control signal) applied to the MR damper based on the interference information. When the input current increases, the magnetic field intensity of the electromagnetic coil inside the damper increases, and thus the shear yield force also increases. Then the generated damping force can be used to mitigate the vehicle vibrations.

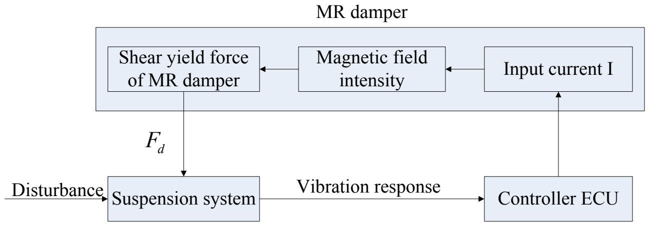

Fig.1 Work process of vehicle suspension system with MR.

It is noted that MR dampers may exist nonsmooth dynamics, e.g. hysteretic, thus accurate modeling of MR damper is essential for its control synthesis. For this purpose, several different dynamic models have been proposed, for instance, Bingham model [11], Bouc-wen model [12], modified Bouc-wen model [13], hyperbolic model [20]. We first review some of these models in terms of the modeling complexity and dynamical behaviors.

Bingham model is the most commonly used MR damper model, which can describe essential characteristics of MR fluid. We can obtain a dynamic equation as [11]:

$$F = f_c \operatorname{sgn}(\dot{x}) + c_0\dot{x} + f_0 \tag{1}$$

where F is the generated damping force, $\dot{x}$ is the piston velocity, $\operatorname{sgn}(\cdot)$ is a signum function. f_c is the friction coefficient associated with the MR fluid, c_0 is the viscosity damping coefficient, and f_0 is the damper force induced by the internal pressure difference of the damper. Bingham model is simple and easy for analysis. It can describe the force-velocity relationship. However, this model assumes that the damper is rigid, and the viscoelastic property of the damper force in the pre-yield region is ignored. Thus, the force-velocity curve may be nonsmooth when the velocity is around zero.

Many researchers further modified the Bingham model by introducing smooth force-velocity curves. The following Bouc-wen model consists of a spring, a viscous damper and a Bouc-wen hysteretic operator [12]. Bouc-wen model can be used to capture the hysteresis behavior of MR dampers, where the damping force is computed as:

$$F = az + c_0\dot{x} + k_0(x - x_0) \tag{2}$$

$$\dot{z} = A\dot{x} - \gamma|\dot{x}|z|z|^{n-1} - \beta\dot{x}|z|^n \tag{3}$$

where c_0 is the viscosity damping coefficient, k_0 is the stiffness coefficient, x_0 is the initial displacement, and a is a constant proportional to the current. z is an auxiliary function that represents the hysteretic component of the MR damper, and γ, β, A are the model parameters that can change the shape of the hysteresis loop and the smoothness within the pre-yield and post-yield regions. The model can be reduced to a common damper if $\alpha = 0$. When $\alpha \neq 0$ the hysteresis characteristics can be described.

Compared to the Bingham model, the curve of the Bouc-wen model is smooth, which can also reflect the nonlinear behavior of MR damper at the low speed regime. However, there are many parameters, which should be calibrated based on the experiment data, i.e. the potential modeling complexity makes it nontrivial for engineering application. Hence, Spencer et al. [13] proposed a modified Bouc-wen model as:

$$F = c_1\dot{y} + k_1(x - x_0) \tag{4}$$

$$\dot{y} = 1/(c_0 + c_1)[az + c_0\dot{x} + k_0(x - y)] \tag{5}$$

$$\dot{z} = A(\dot{x} - \dot{y}) - \gamma|\dot{x} - \dot{y}||z|^{n-1}z - \beta(\dot{x} - \dot{y})|z|^n \tag{6}$$

where c_1 and k_1 are the viscosity coefficient and stiffness coefficient of new damper and spring, respectively. y and z are the auxiliary dynamic variables. This modified Bouc-wen model further improves the accuracy for modeling the exact MR damper behaviors. However, there are two variables y and z that cannot be directly observed, and their physical meaning was not clearly justified. Moreover, the complexity of this model is significant, which may also create difficulties in the modeling phase.

Thus, with the aim to develop a simple, smooth MR model, which is capable to describe hysteretic dynamics, a hyperbolic tangent function can be used to represent the hysteresis characteristics embedded in the MR damper. This is possible by considering the shape and mathematical expressions of tangent functions. Thus linear functions representing the viscous and stiffness together with a tangent function can lead to the following hyperbolic MR model [20]

$$F = F_y z + c_0\dot{x} + k_0 x + f_0 \tag{7}$$

$$z = \tanh\left(\beta\dot{x} + \delta sign(x)\right) \tag{8}$$

where F_y is the dynamic force coefficient associated with the current. z is a hysteretic variable of the hyperbolic tangent function (8), β and δ are the scale factors of hysteretic slope and hysteresis band width, respectively.

Compared to other models, the hyperbolic model contains a simple hyperbolic tangent function, which can be incorporated into the regressor vector for the purpose of online parameter estimation. Thus, this hyperbolic model is suitable for online modeling of MR damper. The online parameter estimation of hyperbolic model will be studied in the following subsection.

2.2 Parameter Estimation for hyperbolic MR damper

As analyzed, the hyperbolic model will be adopted in this paper to describe the nonlinearities and hysteresis of MR damper. According to different applications of MR damper, the associated spring effects f_0 in (7) produced by the internal accumulator may be small or even trivial. Hence, for the ease of a simple analysis, f_0 in the adopted hyperbolic model is neglected [20], thus we have the following model:

$$U = f_I \tanh\left(c_1\dot{z}_{def} + k_1 z_{def}\right) + c_0\dot{z}_{def} + k_0 z_{def} \tag{9}$$

where F_d is the force produced by the damper, c_1, k_1, c_0 and k_0 are appropriate constants; f_I is the dynamic force coefficient associated with the drive current I in the coil of MR damper. The relationship between f_I and the drive current I $\left(0 \le I \le 2\right)$ can be described as [20]:

$$f_I = y_I I \tag{10}$$

where y_I is a constant parameter defining the MR property.

The force-velocity of the hyperbolic model (9)-(10) with different drive current is shown in Fig.2 One may find that the hysteresis loop is clearly indicated. Thus, the hyperbolic model can be used to accurately describe the hysteresis characteristics of MR damper.

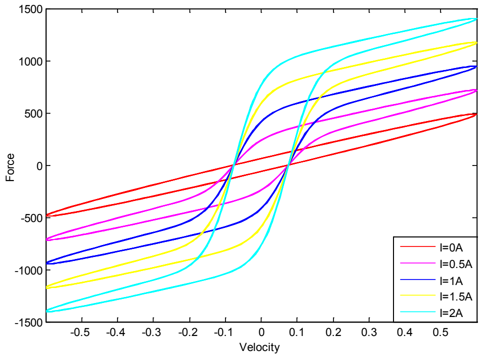

Fig.2 Force-Velocity characteristic of hyperbolic model

Substituting (10) into (9), then F_d can be written as:

$$F_d = y_I I \tanh\left(c_1 \dot{z}_{def} + k_1 z_{def}\right) + c_0 \dot{z}_{def} + k_0 z_{def} = \Phi\theta \quad (11)$$

where $\Phi = [I \tanh(c_1 \dot{z}_{def} + k_1 z_{def}), \dot{z}_{def}, z_{def}]$ is the regressor vector and $\theta = [y_I, c_0, k_0]^T$ is the unknown parameter vector to be estimated.

In [20], a similar hyperbolic model was used to characterize the property of MR damper. However, the parameters y_I, c_0, k_0 are all assumed to be precisely known. In this paper, we will develop an online adaptive framework to estimate y_I, c_0, k_0. For the sake of simplification, the constants c_1, k_1 included in the tangent function are known. Moreover, in this section the MR damper force F_d and the piston velocity $\dot{z}_{def}$ and displacement z_{def} are all accessible or measurable.

To estimate θ in (11) using the damper force F_d, piston velocity $\dot{z}_{def}$ and displacement z_{def}, we will adopt and modify our previous methods presented in [18, 19] to introduce an adaptive law for system (11) with exponential error convergence. Thus, define the auxiliary matrix M and vector N in terms of the following equations:

$$\begin{cases} \dot{M} = -lM + \Phi^T\Phi, \ M(0) = 0 \\ \dot{N} = -lN + \Phi^T F_d, N(0) = 0 \end{cases} \quad (12)$$

where $l > 0$ is a design parameter. As explained in [21], we can obtain M and N by using simple filter operation.

Then another auxiliary vector U can be defined as:

$$U = M\hat{\theta} - N \quad (13)$$

where $\hat{\theta}$ is the estimation of θ, which can be given by the following adaptive law

$$\dot{\hat{\theta}} = -\Gamma U \quad (14)$$

for $\Gamma > 0$ being a constant matrix.

Theorem 1: If the regressor vector Φ defined in the system (11) is persistently excited (PE)[22], the estimation parameter error $\tilde{\theta} = \theta - \hat{\theta}$ exponentially converges to zero.

Proof: It has been shown in [18, 19] that if Φ is PE, then the matrix M in (12) is positive definite, i.e. its minimum eigenvalue $\lambda_{\min}(M) > \sigma > 0$. We select a Lyapunov function as $V = \frac{1}{2}\tilde{\theta}\Gamma^{-1}\tilde{\theta}$. One may calculate $\dot{V}$ as:

$$\dot{V} = \tilde{\theta}^T \Gamma^{-1}\dot{\tilde{\theta}} = \tilde{\theta}^T U = -\tilde{\theta}^T P \tilde{\theta} \le -\mu V \quad (15)$$

where $\mu = 2\sigma / \lambda_{\max}(\Gamma^{-1})$ is a positive constant for all $t > 0$. Then according to Lyapunov's Theorem, the estimation error $\tilde{\theta}$ will converge to zero exponentially.

Remark 1: As shown in the above proof, the variable U used to drive adaptive law (14) contains the information of $\tilde{\theta}$, so that it can attract the estimated parameter $\hat{\theta}$ toward its true value in an exponential manner. Moreover, the observer or predictor used in the traditional methods (e.g. gradient method and RLS approaches [22]) are not needed.

3. Adaptive Estimation and Control

3.1 Quarter-car model

In this paper, a nonlinear quarter-car model shown in Fig.3 is studied, where m_s is the sprung mass, and m_{us} represents the mass of wheel, respectively. F_d and F_s are the force produced by the dampers and springs with the damping coefficient b_e, the stiffening coefficients of linear and nonlinear terms k_s, k_{sn}. F_t and F_b denote the elasticity and damping forces of tire with the stiffness and damping coefficients k_t, b_f. z_s and z_{us} are the displacements of sprung and unsprung masses. z_r is the input of road displacement. U is the control force of the semi-active suspension system, which is generated by hyperbolic model (11).

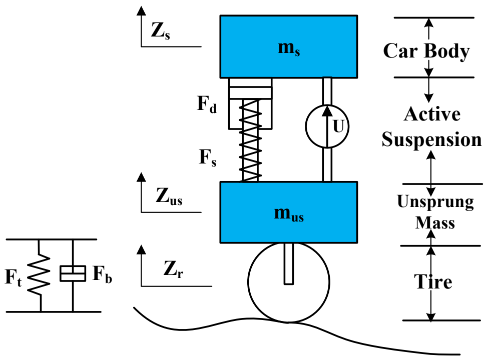

Fig.3 Quarter-car model with semi-active suspension system

According to Newton's second law, the dynamics of the studied suspension system shown in Fig.4 are obtained as [14]:

$$\begin{cases} m_s \ddot{z}_s + F_s + F_d = U \\ m_{us}\ddot{z}_{us} - F_d - F_s + F_t + F_b = -U \end{cases} \quad (16)$$

where the forces are given by $F_s = k_s(z_s - z_{us}) + k_{sn}(z_s - z_{us})^3$. $F_d = b_e(\dot{z}_s - \dot{z}_{us})$. $F_t = k_t(z_{us} - z_r)$. and $F_b = b_f(\dot{z}_{us} - \dot{z}_r)$. To facilitate the control design, we define state variables as:

$$x_1 = z_s, x_2 = \dot{z}_s, x_3 = z_{us}, x_4 = \dot{z}_{us} \quad (17)$$

To incorporate the MR damper into the control design, we substitute (10) into (9), and then the damper output force (9) can be rewritten as follows:

$$U = y_I I \tanh(c_1(\dot{x}_1 - \dot{x}_3) + k_1(x_1 - x_3)) + c_0(\dot{x}_1 - \dot{x}_3) + k_0(x_1 - x_3) \quad (18)$$

Then the system (16) can be rewritten in the state-space form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{1}{m_s}[(c_0 - b_e)(x_2 - x_4) + (k_0 - k_s)(x_1 - x_3) \\ \quad -k_{sn}(x_1 - x_3)^3 + y_I I \tanh(c_1(x_2 - x_4) + k_1(x_1 - x_3))] \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{1}{m_{us}}[(b_e - c_0)(x_2 - x_4) + (k_s - k_0)(x_1 - x_3) + k_{sn}(x_1 - x_3)^3 \\ \quad -k_t(x_3 - z_r) - b_f(x_4 - \dot{z}_r) - y_I I \tanh(c_1(x_2 - x_4) + k_1(x_1 - x_3))] \end{cases}$$

(19)

Other suspension rquirements to be addressed include:

1) *Ride comfort:* This needs to design derive current I which creates the control force U to regulate the vertical displacement of vehicle body under the road shocks, i.e. $x_1 = z_s \to 0$.

2) *Road holding*: The firm uninterrupted contact of wheels to road should be ensured for the safety of passengers, that is

$$\left|F_t\right| < (m_s + m_{us})g \tag{20}$$

3) *Suspension movement limitation*: The suspension space should not exceed the allowable maximum due to the used mechanical structures, i.e. the difference $z_s - z_{us}$ should be bounded by the maximum suspension space $z_{\max}$ as:

$$\left|z_s - z_{us}\right| \le z_{\max} \tag{21}$$

The suspension performance 1), 2) and 3) will be studied by introducing an adaptive control, where the unknown parameters will be also online estimated.

3.2 Adaptive Control Design

To address the regulation of vertical displacement of system (19), we first define the filtered error variable as:

$$s_1 = [\Lambda, 1][x_1, x_2]^T \tag{22}$$

where $\Lambda > 0$ is a positive constant. Thus, s_1 is bounded as long as the filtered error s_1 is bounded. In particular, $\left|x_1\right| \le \left|s_1\right| / \Lambda$ and $\left|x_2\right| \le 2\left|s_1\right|$ are true for zero initial condition.

Furthermore, we can obtain the time derivate of s_1 as:

$$\begin{aligned}\dot{s}_1 = \Lambda x_2 + \frac{1}{m_s}[&(c_0 - b_e)(x_2 - x_4) + (k_0 - k_s)(x_1 - x_3) \\ &- k_{sn}(x_1 - x_3)^3 + y_I I \tanh\left(c_1(x_2 - x_4) + k_1(x_1 - x_3)\right)]\end{aligned} \tag{23}$$

In this paper, the coefficients of springs, the mass of car body and the parameters of hyperbolic model are all unknown. We will present an online estimation algorithm to address the unknown dynamics. Hence, we denote the system dynamics as a more compact form as

$$\begin{aligned}T(Z) &= 1/m_s\left[(c_0 - b_e)(x_2 - x_4) + (k_0 - k_s)(x_1 - x_3) - k_{sn}(x_1 - x_3)^3\right] \\ &= W_1^T \phi_1(Z_1)\end{aligned} \tag{24}$$

where $W_1 = \left[(c_0 - b_e)/m_s, (k_0 - k_s)/m_s, k_{sn}/m_s\right]^T$ is the parameter vector to be estimated, $\phi_1(Z_1) = \left[x_1 - x_3, x_2 - x_4, (x_1 - x_3)^3\right]^T$ is the regressor vector with $Z_1 = [x_1, x_2, x_3, x_4] \in R^4$.

Substituting (24) into (23), then $\dot{s}_1$ can be written as:

$$\dot{s}_1 = \Lambda x_2 + \Theta^T \Psi \tag{25}$$

where $\Theta = [W_1^T, W_2^T]^T$ is the augmented parameter vector and $\Psi = [\phi_1^T(Z_1), \phi_2^T(Z_2) I]^T$ is the augmented regression vector with $W_2 = -y_I / m_s$ and $\phi_2(Z_2) = \tanh(c_1(x_2 - x_4) + k_1(x_1 - x_3))$.

We denote $\hat{\Theta} = [\hat{W}_1^T, \hat{W}_2^T]^T$ as the estimation of unknown parameter Θ and then the drive current I can be designed as:

$$I = \frac{1}{\hat{W}_2^T \phi_2(Z_2)}\left[-\hat{W}_1^T \phi_1(Z_1) - Ks - \Lambda x_2\right] \tag{26}$$

where $K > 0$ is the feedback gain, $\hat{W}_1, \hat{W}_2$ are the estimation of W_1, W_2, which will be updated based on the adaptive law given in (32).

Define the filtered variables s_{1f}, Ψ_f, x_{2f} of s_1, Ψ, x_2 as:

$$\begin{cases} k\dot{s}_{1f} + s_{1f} = s_1, & s_f(0) = 0 \\ k\dot{\Psi}_f + \Psi_f = \Psi, & \Psi_f(0) = 0 \\ k\dot{x}_{2f} + x_{2f} = x_2, & x_{2f}(0) = 0 \end{cases} \tag{27}$$

where $k > 0$ is a scalar filter parameter.

According to (25) and (27), one can obtain that:

$$\dot{s}_{1f} = \frac{s_1 - s_{1f}}{k} = \Lambda x_{2f} + \Theta^T \Psi_f \tag{28}$$

Moreover, we define the auxiliary matrix M_1 and vector N_1 in terms of following filter operations:

$$\begin{cases} \dot{M}_1 = -lM_1 + \Psi_f \Psi_f^T, & M_1(0) = 0 \\ \dot{N}_1 = -lN_1 + \Psi_f[(s_1 - s_{1f})/k - \Lambda x_{2f}], & N_1(0) = 0 \end{cases} \tag{29}$$

where $l > 0$ is a design parameter.

Then another vector U_1 can be obtained based on M_1, N_1 as:

$$U_1 = M_1 \hat{\Theta} - N_1 \tag{30}$$

The adaptive law for updating $\hat{\Theta}$ is given by:

$$\dot{\hat{\Theta}} = \Gamma_1 s_1 \Psi - \Gamma_1 \kappa U_1 \tag{31}$$

where $\Gamma_1 > 0$ is a constant diagonal matrix and $\kappa > 0$ is a constant scalar.

Theorem 2: For system (19) with control (26) and (31), if the regressor vector Ψ in (25) is PE, then the control error s_1 and estimation error $\tilde{\Theta}$ exponentially to zero.

Proof: As proved in [21], if Ψ in (25) is PE, the minimum eigenvalue of the matrix P_1 fulfills $\lambda_{\min}(M_1) > \sigma_1 > 0$. By substituting (26) into (25), $\dot{s}_1$ can be written as:

$$\dot{s}_1 = -Ks_1 + \tilde{\Theta}^T \Psi \tag{32}$$

On the other hand, according to (28)~(30), the vector U_1 defined in (30) is equivalent to $U_1 = -M_1 \tilde{\Theta}$ as shown in [21]. Therefore, we select a Lyapunov function as:

$$V_1 = \frac{1}{2}s_1^2 + \frac{1}{2}\tilde{\Theta}^T \Gamma_1^{-1} \tilde{\Theta} \tag{33}$$

Then the time derivate of V can be obtained as:

$$\dot{V}_1 = s_1 \dot{s}_1 + \tilde{\Theta}^T \Gamma_1^{-1} \dot{\tilde{\Theta}} = -Ks_1^2 - \kappa \tilde{\Theta}^T P_1 \tilde{\Theta} \le -\mu_1 V_1 \tag{34}$$

where $\mu_1 = \min\left\{2K, 2\kappa\sigma_1 / \lambda_{\max}\left(\Gamma_1^{-1}\right)\right\}$ is a positive constant. According to Lyapunov's Theorem, the control eror s_1 and estimation error $\tilde{\Theta}$ all converge to zero exponentially, where the convergence rate depends on the control gain K, the excitation level σ_1 and the leaning gains o and Γ_1.

Remark 2: The use of the leakage term $\Gamma_1 \kappa U_1$ in adaptive law (31) is inspired by our previous work [18, 19] and [21]. As shown in the above proof, the inclusion of variable U_1 leads to a quadratic term (i.e. $\tilde{\Theta}^T P_1 \tilde{\Theta}$) of the estimation error $\tilde{\Theta}$ in the Lyapunov analysis. Thus the estimated parameter can converge to its true values in an exponential manner. This could also improve the suspension control performance.

3.3 Suspension performance analysis

The convergence of x_1 have been guaranteed in Section 3.2. In the following, we will address the stability of overall system, and another two suspension performance requirements (20) and (21).

First, the boundedness of the state variables x_3, x_4 of system (19) is studied. Thus, substituting (26) into (19), one can obtain the following dynamics:

$$\dot{x} = Ax + \omega \tag{35}$$

where

$$x = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}, A = \begin{bmatrix} 0 & 1 \\ -\dfrac{k_t}{m_{us}} & -\dfrac{b_f}{m_{us}} \end{bmatrix} \tag{36}$$

$$\omega = \begin{bmatrix} 0 \\ \frac{k_t}{m_{us}} z_r + \frac{b_f}{m_{us}} \dot{z}_r + \frac{m_s}{m_{us}} \omega_1 \end{bmatrix} \tag{37}$$

where $\omega_1 = Ks_1 + \Lambda x_2 - \tilde{\Theta}^T \Psi$ denotes the effects of the residual error, which is bounded because s_1, x_2 and $\tilde{\Theta}$ are all bounded. Therefore, ω is bounded, i.e. $\|\omega\| \le \varpi$ holds for a positive constant $\varpi > 0$.

Since A defined in (36) is stable, there exist positive matrices P, Q so that the Lyapunov equation $A^T P + AP = -Q$ holds. We select a Lyapunov function as $V = x^T Px$, then

$$\dot{V} \le -[\lambda_{\min}(Q) - \frac{1}{\eta} \lambda_{\max}(P)] \|x\|^2 + \eta \lambda_{\max}(P) \varpi^2 \tag{38}$$

Then for appropriately designed parameters fulfilling $\eta > \lambda_{\max}(P)/\lambda_{\min}(Q)$, it follows from (38) that:

$$\dot{V} \le -\alpha V + \beta \tag{39}$$

where $\alpha = [\lambda_{\min}(Q) - \lambda_{\max}(P)/\eta]/\lambda_{\min}(P)$ and $\beta = \eta \lambda_{\max}(P) \varpi^2$ are all positive constants. This implies that the state variables x_3, x_4 are all bounded by:

$$|x_i| \le \sqrt{(V(0) + \beta/\alpha)/\lambda_{\min}(P)}, i = 3,4 \tag{40}$$

So that the bound of the tire load can be calculated as:

$$|F_t + F_b| \le k_t \sqrt{(V(0) + \beta/\alpha)/\lambda_{\min}(P)} + k_t |z_r| + b_f |\dot{z}_r| \tag{41}$$

Then the parameters η and P can be appropriately selected, such that the performance requirement of the road holding (21) can be guaranteed.

Finally, we can obtain the bounds of suspension spaces as:

$$|x_1 - x_3| \le \sqrt{2V_1}/\Lambda + \sqrt{(V(0) + \beta/\alpha)/\lambda_{\min}(P)} \le z_{\max} \tag{42}$$

It is clear that the suspension movement limitation (21) can be fulfilled if the parameters $\Lambda, K, o, \Gamma_1, \sigma_1, \eta, P$ are designed appropriately.

4. Simulations

In this section, numerical simulations are provided to illustrate the effectiveness of the proposed algorithms. The parameters of the MR damper and quarter-car model are given as: $m_s = 600kg, m_{us} = 60kg, k_s = 18000N/m, k_{sn} = 1000N/m$, $k_t = 200000N/m, b_f = 1000Ns/m, b_e = 2500Ns/m$, $b_c = 2200Ns/m$, $c_0 = 810.78Ns/m, c_1 = 13.76s/m, y_I = 457.04N/A$, $k_1 = 10.54\ 1/m$, $k_0 = 620.79N/m, z_{\max} = 0.15m$. The following two cases are simulated:

Case 1 (Adaptive parameter estimation of MR damper): In this simulation, only the MR damper dynamics (11) is considered to show the online modeling method (14). Thus, we set the velocity of piston as $\dot{z}_{def} = 0.6\cos(6t)$ and the input current as $I = 2$. The estimation performance of the gradient method and the proposed method are compared. For fair comparison, the initial simulation conditions are set as $\theta(0) = [0\ 0\ 0.001]^T$. The simulation parameters are set as $\Gamma = 30diag[0.065\ \ 0.53\ \ 6.4]$ and $k = 0.001, l = 1$. On may find from Fig.4 that the velocity-force curves with the estimated parameters are comparable to its nominal counterparts. It is clearly shown that the estimated model via (14) can capture the essential dynamics of the realistic MR damper. This implies that the estimated parameters converge to their true values. Thus, the proposed novel leakage term contributes to improve the parameter estimation performance.

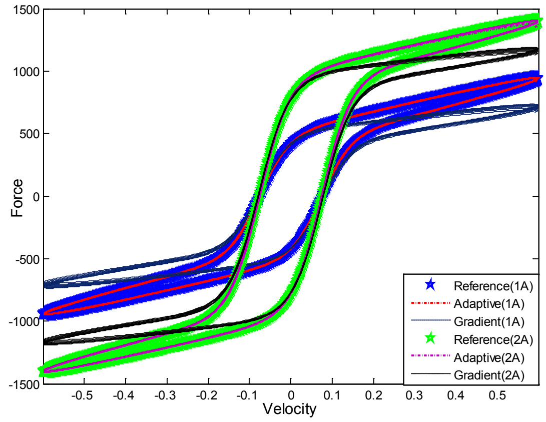

Fig. 4 Comparative performance of velocity-force characteristic with different estimation methods.

Case 2 (Adaptive control with sinusoid road displacement): The proposed control and estimation are finally validated. In this case, the road disturbance is given as follows:

$$z_r = \begin{cases} \frac{h}{2}\left(1 - \cos\left(\frac{2\pi V_s}{l} t\right)\right), & 0 \le t \le \frac{l}{V_s} \\ 0 & t \ge \frac{l}{V_s} \end{cases} \tag{43}$$

where b=0.1$m, l = 5m$ are the height and the length of the bump road profile, and $V_s = 45km/h$ is the vehicle velocity. The suspension performance of the proposed semi-active suspension methods (26) and (31) are compared to passive suspension system (i.e. $I = 0$) under the initial values $x_1(0) = 0.01m, x_i(0) = 0, i = 2,3,4$, $\Theta(0) = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0.5]^T$. The proposed control and adaptive law are simulated with parameters $K = 40$, $\Lambda = 5$, $k = 0.001, l = 1$ $\kappa = 1/3$ and $\Gamma_1 = 30diag([0.14\ \ 3.873\ \ 0\ \ 0\ \ 0.02]^T)$. Simulation results of the vertical vehicle displacement x_1 are given in Fig.5. Compared with passive suspension and semi-active control with gradient adaptation (i.e. $\kappa = 0$ in (31)), the proposed control for semi-active suspension system has lower peak than others and thus diminishes the vertical displacement effectively. The parameter estimation performance (i.e. $\hat{\Theta}$) is given in Fig.6. From Fig.6, we can find that the proposed adaptive law can estimate the unknown parameters. However, the gradient method cannot guarantee satisfactory parameter estimation convergence although the steady-state control performance can be achieved. In addition, the other two suspension performances (20) and (21) can be fulfilled as shown in Fig.7 and Fig.8.

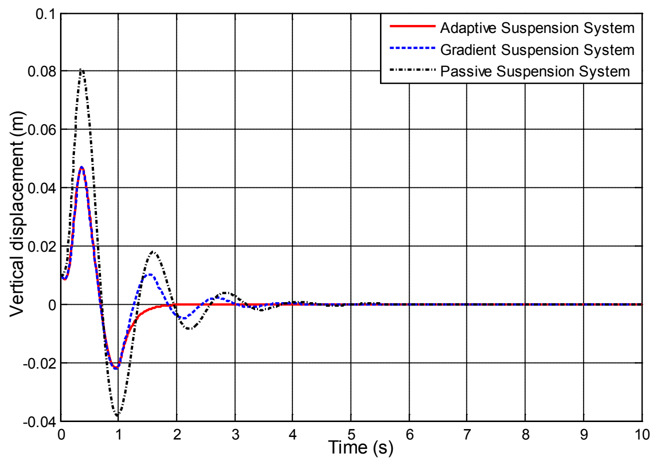

Fig.5 Comparative performance of vertical displacements

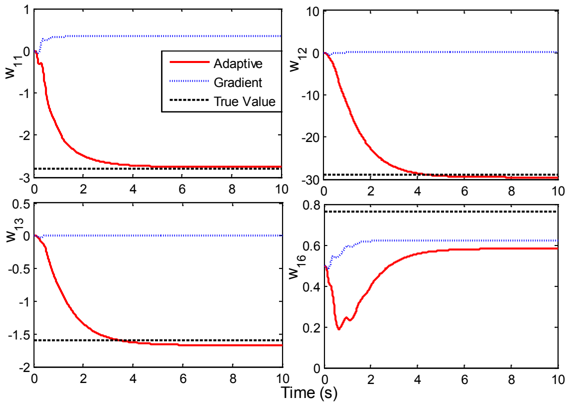

Fig.6 Parameter estimation of Θ.

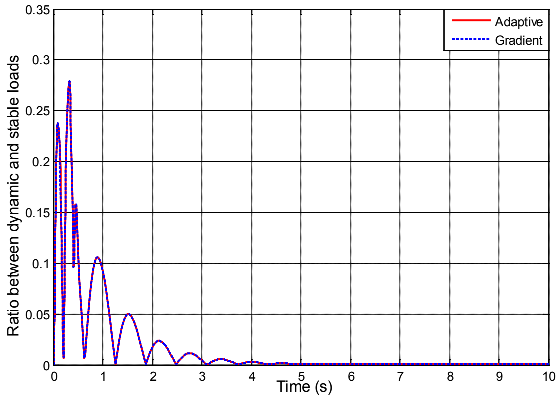

Fig.7 Dynamic tire load of semi-active suspension systems

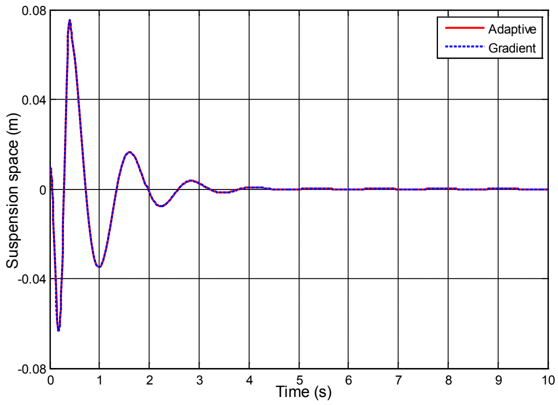

Fig.8 Suspension space of semi-active suspension systems

5. Conclusions

In this paper, an adaptive estimation and control for vehicle semi-active suspension system with magneto-rheological (MR) damper is proposed. The unknown parameters of a simple MR damper model are estimated by using a recently introduced adaptive algorithm based on the estimation error. In this sense, online mathematical modeling of MR damper can be obtained, where only measurable hysteretic variable and damper force are used. Moreover, the MR damper is further incorporated into the vehicle suspension system, and an adaptive control is developed to regulate the vertical displacement of vehicle body. Simultaneous suspension and parameter estimation can be achieved by introducing a leakage term of the estimation error in the adaptive law. The suspension requirements of ride comfort and vehicle safety are also studied. The proposed approaches are validated by comparative simulations based on a quarter-car model.

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